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The incompleteness of Peircean semiotics at the hand of intervals of natural numbers

1. That the Peirce-Bensean semiotic with its 10 sign classes, their 10 dual reality thematics are incomplete, was already shown in a number of article that are to be found in my book "Semiotic Structures and Processes" (Toth 2008). There two restriction laws that cause this incompleteness:

1.1. The Law of Triadicity

This means that a sign relation has to consist 1. of three relations x, y z and that 2. x, y, z have to be mapped on the fundamental categories 1, 2, 3 so that x, y and z are pairewise different. Therefore, semiotic relations like (3.a 3.b 1.c) or (2.b 2.b 2.b) are not considered sign classes. Note that this law does not exclude sign classes of the forms (3.a 1.c 2.b), (2.b 3.a 1.c), (2.b 1.c 3.a), (1.c 3.a 2.b), and (1.c 2.b 3.a) besides the "canonical" Peircean form (3.a 2.b 1.c), cf. Toth (2008, pp. 177 ss.).

1.2. The Law of Trichotomic Inclusion

This second restriction excludes sign relations like (3.3 2.2 1.1) or generally sign relations whose trichotomic values are smaller than the trichotomic values of the sub-sign before: (3.a 2.b 1.c) with ($a \leq b \leq c$). This law thus reduces the possible total of $3^3 = 27$ to 10 sign classes.

In Toth (2009), 1.) We had mapped the sub-signs of a 3-contextural 3×3 matrix to the first 3 contextures of qualitative numbers. 2.) We have given the decimal correspondences to these qualitative (proto-, deutero- and trito-) numbers:

Proto	Deutero		Trito	Deci	
		(1.1), (1.2),			
0	0	(2.1), (2.2)	0	0	C1
		(2.2), (2.3),			
00	00	(3.2), (3.3)	00	0	
01	01		01	1	C2
		(1.1), (1.3),			
000	000	(3.1), (3.3)	000	0	
001	001		001	1	
012	012		010	3	C3
			011	4	
			012	5	

In doing so, it was possible to determine the 10 sign classes and their dual reality thematics by intervals of Peano numbers over the qualitative numbers which correspond to the sub-signs as in the above table.

$$(3.1_3\ 2.1_1\ 1.1_{1,3}) \times (1.1_{3,1}\ 1.2_1\ 1.3_3) \rightarrow I = [[0, 5], [0], [0, 5]]$$
$$(3.1_3\ 2.1_1\ 1.2_1) \times (2.1_1\ 1.2_1\ 1.3_3) \rightarrow I = [[0, 5, [0], [0]]$$
$$(3.1_3\ 2.1_1\ 1.3_3) \times (3.1_3\ 1.2_1\ 1.3_3) \rightarrow I = [[0, 5], [0], [0, 5]]$$
$$(3.1_3\ 2.2_{1,2}\ 1.2_1) \times (2.1_1\ 2.2_{2,1}\ 1.3_3) \rightarrow I = [[0, 5], [0, 1], [0]]$$
$$(3.1_3\ 2.2_{1,2}\ 1.3_3) \times (3.1_3\ 2.2_{2,1}\ 1.3_3) \rightarrow I = [[0, 5], [0, 1], [0, 5]]$$
$$(3.1_3\ 2.3_2\ 1.3_3) \times (3.1_3\ 3.2_2\ 1.3_3) \rightarrow I = [[0, 5], [0, 1], [0, 5]]$$
$$(3.2_2\ 2.2_{1,2}\ 1.2_1) \times (2.1_1\ 2.2_{2,1}\ 2.3_2) \rightarrow I = [[0, 1], [0, 1], [0]]$$
$$(3.2_2\ 2.2_{1,2}\ 1.3_3) \times (3.1_3\ 2.2_{2,1}\ 2.3_2) \rightarrow I = [[0, 5], [0, 1], [0, 5]]$$
$$(3.2_2\ 2.3_2\ 1.3_3) \times (3.1_3\ 3.2_2\ 2.3_2) \rightarrow I = [0, 5], |0, 1], [0, 5]]$$
$$(3.3_{2,3}\ 2.3_2\ 1.3_3) \times (3.1_3\ 3.2_2\ 3.3_{3,2}) \rightarrow I = [[0, 5], [0, 1], [0, 5]$$

The order of the intervals is as follows. We also introduce capitals in order to determine the types of intervals which we will look at after:

$$(3.2_2\ 2.2_{1,2}\ 1.2_1) \times (2.1_1\ 2.2_{2,1}\ 2.3_2) \rightarrow \quad I = [[0, 1], [0, 1], [0]] \qquad \text{AAB}$$

$$(3.1_3\ 2.1_1\ 1.2_1) \times (2.1_1\ 1.2_1\ 1.3_3) \rightarrow \quad I = [[0, 5], [0], [0]] \qquad \text{CBB}$$

$$\left.\begin{array}{lll}
(3.1_3\ 2.1_1\ 1.1_{1,3}) \times (1.1_{3,1}\ 1.2_1\ 1.3_3) & \rightarrow & I = [[0, 5], [0], [0, 5]] \\
(3.1_3\ 2.1_1\ 1.3_3) \times (3.1_3\ 1.2_1\ 1.3_3) & \rightarrow & I = [[0, 5], [0], [0, 5]]
\end{array}\right\} \text{CBC}$$

$$(3.1_3\ 2.2_{1,2}\ 1.2_1) \times (2.1_1\ 2.2_{2,1}\ 1.3_3) \quad \rightarrow \quad I = [[0, 5], [0, 1], [0]] \qquad \text{CAB}$$

$$\left.\begin{array}{lll}
(3.1_3\ 2.2_{1,2}\ 1.3_3) \times (3.1_3\ 2.2_{2,1}\ 1.3_3) & \rightarrow & I = [[0, 5], [0, 1], [0, 5]] \\
(3.1_3\ 2.3_2\ 1.3_3) \times (3.1_3\ 3.2_2\ 1.3_3) & \rightarrow & I = [[0, 5], [0, 1], [0, 5]] \\
(3.2_2\ 2.2_{1,2}\ 1.3_3) \times (3.1_3\ 2.2_{2,1}\ 2.3_2) & \rightarrow & I = [[0, 5], [0, 1], [0, 5]] \\
(3.2_2\ 2.3_2\ 1.3_3) \times (3.1_3\ 3.2_2\ 2.3_2) & \rightarrow & I = [[0, 5], [0, 1], [0, 5]] \\
(3.3_{2,3}\ 2.3_2\ 1.3_3) \times (3.1_3\ 3.2_2\ 3.3_{3,2}) & \rightarrow & I = [[0, 5], [0, 1], [0, 5]]
\end{array}\right\} \text{CAC}$$

First, one recognizes that the 10 sign classes are divided in only 5 classes according to their intervals of Peano numbers which are equivalents to the qualitative numbers corresponding to their contextures.

Second, looking at the types AAB, CBB, CBC, CAB, and CAC, we can easily find the lacking types:

AAB	(BBC)	(CCA)	(CCB)	(ABC)
(ABA)	(BCB)	CAC	CBC	(ACB)
(BAA)	CBB	(ACC)	(BCC)	(BAC)
				(BCA)
				CAB
				(CBA)

The types in parenthesis are minimal. E.g., in analogy to AAB and CCA, there could be also stipulated a type CCB, etc. However, if we start with the above 5 types and their permutations, then we find in the 10 sign classes and their reality thematics actually 5/18 types, i.e. less then 1/3. But let us now look at the lacking types of intervals:

AAB = [[0, 1], [0, 1], [0]] → ABA = [[0, 1], [0], [0, 1]], BAA = [[0], [0, 1], [0, 1]]
CBB = [[0, 5], [0], [0]] → BBC = [[0], [0], [0, 5]], BCB = [[0], [0, 5], [0]]
CAC = [[0, 5], [0, 1], [0, 5]] → CCA = [[0, 5], [0, 5], [0, 1]],
ACC = [[0, 1], [0, 5], [0, 5]]
CBC = [[0, 5], [0], [0, 5]] → CCB = [[0, 5], [0, 5], [0]], BCC = [[0], [0, 5], [0, 5]]
CAB = [[0, 5], [0, 1], [0]] → ABC = [[0, 1], [0], [0, 5]], ACB = [[0, 1], [0, 5], [0]],
BAC = [[0], [0, 1], [0, 5]], BCA = [[0], [0, 5], [0, 1]],
CAB = [[0, 5], [0, 1], [0]], CBA = [[0, 5], [0], [0, 1]]

We can now try to reconstruct sign relations that are lacking in the Peirce-Bensean system, however, in the multi-ordinal possibility given by the ambiguousness of the qualitative numbers which have been mapped to the Peano numbers in our intervals:

ABA = [[0, 1], [0], [0, 1]]	=	e.g., (3.2 2.2 2.3)
BAA = [[0], [0, 1], [0, 1]]	=	e.g., (2.2 3.2 3.3)
BBC = [[0], [0], [0, 5]]	=	e.g., (2.2, 2.2, 3.1)
BCB = [[0], [0, 5], [0]	=	e.g., (2.2, 3.1, 2.2)
CCA = [[0, 5], [0, 5], [0, 1]]	=	e.g., (3.1, 3.1, 2.3)
ACC = [[0, 1], [0, 5], [0, 5]]	=	e.g., (2.3, 3.1, 3.1)
CCB = [[0, 5], [0, 5], [0]]	=	e.g., (3.1, 3.1, 2.2)
BCC = [[0], [0, 5], [0, 5]]	=	e.g., (2.2, 3.1, 3.1)
ABC = [[0, 1], [0], [0, 5]]	=	e.g., (3.2, 2.2, 3.1)
ACB = [[0, 1], [0, 5], [0]]	=	e.g., (3.2, 3.1, 2.2)
BAC = [[0], [0, 1], [0, 5]]	=	e.g., (2.2, 3.2, 3.1)
BCA = [[0], [0, 5], [0, 1]]	=	e.g., (2.2, 3.1, 3.2)
CAB = [[0, 5], [0, 1], [0]]	=	e.g., (3.1, 3.2, 2.2)
CBA = [[0, 5], [0], [0, 1]]	=	e.g., (3.1, 2.2, 3.2)

Although it is possible also to (re-)construct sign classes which obey the Law of Triadicity, we have given here choices in which this law is violated in order to show that all these types could never be realized without the abolishing of this law. Let us sum up: The abolishment of the Law of Trichotomic Inclusion enables to reach all the combinatory possible $3^3 = 27$ 3-adic sign relation. The abolisment of the Law of Triadicity enables to reach all the combinatory possible $9^3 = 243$ possible 3-adic sign relation whose members have not to be anymore pairwise different. The analysis of the intervals in this work therefore leads to exactly the same results as our former studies about sub-signs and semioses.

Bibliography

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